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ROTATION OF THE HORIZONTAL VELOCITY VECTOR IN THE ATMOSPHERIC TIDES

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ABSTRACT. We have studied the direction of rotation of the horizontal velocity vector in the atmosphere tides above 80 km. We show that there is no characteristic rotational direction associated with each hemisphere but that there is for each tidal mode, zones of latitude within which one can unambiguously define the rotational direction. This result, which comes from treatment of the tidal equations (without any hypotheses concerning the solution of the radial equation) is valid from ground level in the case of time related rotation.

Introduction

We acknowledge, generally, that the horizontal velocity vector for the different atmospheric tidal modes turns in the direction of the hands of a watch (clockwise direction) in the Northern Hemisphere and in an opposite direction in the Southern Hemisphere, and that it rotates as a function of time about a fixed point in space or rotates as a function of altitude at a given instant. In this latter case it is necessary that in the zone considered the vertical flow of energy should be positive. /1*

In the study of the rotation of the velocity vector, as in all those domains which have a connection with tidal theory, it is necessary to distinguish two problems: the variation of parameters in relation to the latitude and their variation in relation to altitude. The first problem can be treated in the same manner for all the systems similar to that of a planet which turns within an atmosphere, without taking into account the thermal structure of the latter. For the second problem it is necessary to take into account not only the thermal

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1. Aeronautic Service of C.N.R.S., 91 - Verrieres-le-Buisson.

* Numbers in the margin indicate pagination in the foreign text.

structure of the atmosphere, but also the form of the excitation functions which in the case of thermal force (energy) is not yet completely known.

This is why we have made the analysis without making any hypothesis about the form of the radial function. In particular, we are going to show that oscillation modes exist for which the rotation is produced opposite to the direction usually acknowledged in some zones of latitude and which we will call reverse rotation zones.

1. Spatial Rotation /2

Although the tidal equations are very well understood, we are going to write those which we need in order to avoid all notation ambiguities.

If we reduce to a single oscillation mode the horizontal components of velocity, they can be written in the following manner [Siebert, 1961]:

$$u_n = \frac{\gamma g h_n e^{x/2}}{4a^2 \omega^2 (f^2 - \cos^2\theta)} \left[\frac{dy_n(x)}{dx} - \frac{1}{2} y_n(x)\right] \left[\frac{d}{d\theta} + \frac{s}{f}\cos\theta\right] \Theta^s_{\lambda,n}(\theta) e^{i(\sigma t + s\varphi)} \quad (1)$$

$$v_n = \frac{i\gamma g h_n e^{x/2}}{4a^2 \omega^2 (f^2 - \cos^2\theta)} \left[\frac{dy_n(x)}{dx} - \frac{1}{2} y_n(x)\right] \left[\frac{\cos\theta}{f}\frac{d}{d\theta} + \frac{s}{\sin\theta}\right] \Theta^s_{\lambda,n}(\theta) e^{i(\sigma t + s\varphi)} \quad (2)$$

where

ω = angular velocity of rotation in relation to the fixed sphere;

a = earth's radius;

σ = angular frequency of oscillation;

f = $\sigma/2\omega$;

γ = adiabatic constant;

h_n = equivalent height of oscillation;

g = acceleration of gravity;

θ = colatitude (0 at the North Pole);

φ = longitude;

s = whole number which indicates the periodicity in relation to φ;

λ = whole number bound to σ by the following equality, $\lambda = \sigma T/2\pi$, where T equals the duration of a mean solar or lunar day.

x = reduced height defined by

$$x = \int_0^z \frac{d\xi}{H(\xi)} \tag{3}$$

where

z = altitude above sea level;

ξ = integration parameter;

H = scale height (reference height).

Velocities u_n and v_n are defined positively towards the south and east, respectively;

$\Theta^s_{\lambda,n}$ is the Hough function;

$y_n(x)$ is the radial equation solution.

$$\frac{d^2y_n(x)}{dx} + \left[\frac{1}{h_n}\left(\varkappa H(x) + \frac{dH(x)}{dx}\right) - \frac{1}{4}\right] y_n(x) = \frac{\varkappa J_n(x)\, e^{-x/2}}{\gamma g h_n}$$

$$\varkappa = \frac{\gamma - 1}{\gamma} \tag{4}$$

and $J_n(x)$ is the component variation, as a function of time, of the quantity of heat per unit mass producing the oscillation. In the manner in which the equations at (1) and (2) have been written, $y_n(x)$ is a complex function of the real variable. Since we are only considering one mode, n, we omit the suffix n and write:

$$y(x) = y_R(x) + i\, y_I(x) \tag{5}$$

$$Y_R(x) = \frac{dy_R(x)}{dx} - \frac{1}{2} y_R(x) \tag{6}$$

$$Y_I(x) = \frac{dy_I(x)}{dx} - \frac{1}{2} y_I(x) \tag{7}$$

If we make:

$$\gamma(x) = \text{arc tg}\, \frac{Y_I(x)}{Y_R(x)} \tag{8}$$

then the equations at (1) and (2) can be written,

$$v = -Ae^{x/2}\,|Y(x)|\,v(\theta)\sin\left[\sigma t + s\varphi + \gamma(x)\right] \tag{9}$$

$$u = Ae^{x/2}\,|Y(x)|\,u(\theta)\cos\left[\sigma t + s\varphi + \gamma(x)\right] \tag{10}$$

where

$$A = \frac{\gamma g h}{4a\omega^2} \tag{11}$$

$$v(\theta) = \frac{1}{(f^2 - \cos^2\theta)}\left[\frac{\cos\theta}{f}\frac{d}{d\theta} + \frac{s}{\sin\theta}\right]\Theta(\theta) \tag{12}$$

$$u(\theta) = \frac{1}{(f^2 - \cos^2\theta)}\left[\frac{d}{d\theta} + \frac{s}{f}\cos\theta\right]\Theta(\theta) \tag{13}$$

We will call α the angle the horizontal velocity makes with the north-south axis:

$$\alpha = \operatorname{arc\,tg}\left[-f(\theta)\,\operatorname{tg}\left[\sigma t + s\varphi + \gamma(x)\right]\right] \tag{14}$$

where

$$f(\theta) = \frac{v(\theta)}{u(\theta)} \tag{15}$$

The direction of rotation in relation to the altitude is given by the sign of the expression:

$$\frac{d\,\alpha(x)}{dx} = \frac{-f(\theta)}{\left[1 + [-f(\theta)\,\operatorname{tg}(\sigma t + s\varphi + \gamma(x)]^2\right]\cos^2\left[\sigma t + s\varphi + \gamma(x)\right]} \cdot \frac{\dfrac{dY_I(x)}{dx}Y_R(x) - \dfrac{dY_R(x)}{dx}Y_I(x)}{|Y|^2} \tag{16}$$

that is to say by the signs of $f(\theta)$ and

$$\frac{dY_I(x)}{dx}Y_R(x) - \frac{dY_R(x)}{dx}Y_I(x) \tag{17}$$

We acknowledge that it is a question of a region where the heat producing the tides is negligible. In this case the equation at (4) becomes homogeneous. Of course, this equation is homogeneous for the gravitational tides everywhere /3 and we can write:

$$\frac{d^2y(x)}{dx^2} + K^2(x)\,y(x) = 0 \tag{18}$$

where

$$K(x) = \left[\frac{1}{h}\left(H(x) + \frac{dH(x)}{dx}\right) - \frac{1}{4}\right]^{1/2} \tag{19}$$

Using the equations at (5), (6), (7), (18) and (19), expression (17) can be written:

$$\left[K^2(x) + \frac{1}{4}\right] Im\left[y^*(x)\,\frac{dy(x)}{dx}\right] \tag{20}$$

(Im = imaginary part, and * complex conjugation)

The first term in (20) is positive, unless we assume a very large negative drop in temperature.

The second term is also positive when the energy is propagated upwards.

It is necessary to add that the complete expression of the vertical energy flux [Wilkes, 1949] is:

$$W = \frac{1}{2}\left[\frac{P(0)h\,\gamma^2}{\sigma}\right]\left(\Theta(0)\right)^2 Im\left[y^*(x)\,\frac{dy(x)}{dx}\right] \tag{21}$$

For equivalent negative heights we find that in the diurnal tide [Lindzen, 1966; Kato, 1967), the second term in (20) should be negative so that we can have a positive energy flux.

It is easy to see that, in this case, expression (17) is also positive.

We arrive at the conclusion that the direction of rotation of the velocity vector depends solely on the sign of $f(\theta)$.

The equation at (16) shows that when $f(\theta) > 0$, the rotation is in a clockwise direction (Northern Hemisphere), but in the reverse direction when $f(\theta) < 0$; that is, in the latitudes where expressions (12) and (13) have the opposite sign.

If $f(\theta)$ is zero or infinity, that is $v(\theta) = 0$ or $u(\theta) = 0$, then expression (16) is cancelled from that which was expected from (9) and (10). This states that for corresponding latitudes the oscillation is polarized in a plane either N-S or E-W. In other words, some reverse rotation zones are always bounded by latitudes where the oscillation is polarized linearly.

The computation of (12) and (13) shows that several modes exist for which $f(\theta) < 0$, for certain zones of latitude. We see this from the examples in Figures 1, 2, 3 and 4.

For the semidiurnal tide, we see that the mode $S^2_{2.4}$ has a reverse rotation zone between 23° and 30° of latitude. $S^2_{2.6}$ and $S^2_{2.8}$ also have reverse rotation zones, but they are shorter. In the terdiurnal tide, the phenomenon is most

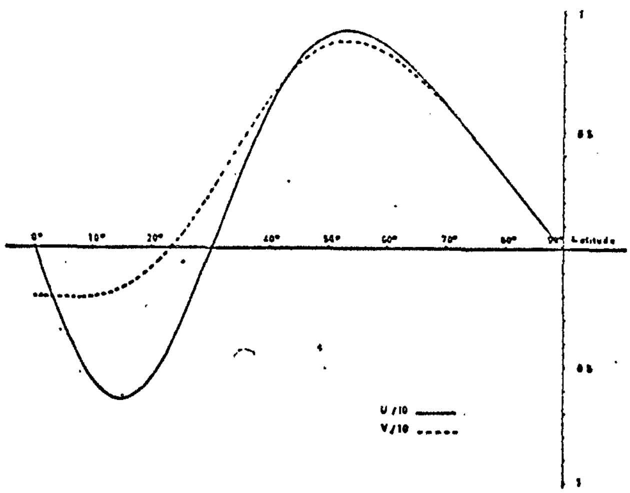

Figure 1.

Variation in latitude of v and u for the second mode of the semidiurnal symmetrical tide.

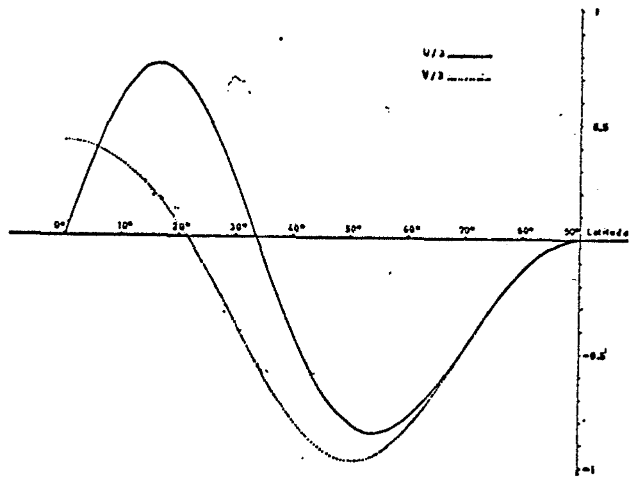

Figure 2.

Variation in latitude of v and u for the second mode of the terdiurnal symmetrical tide.

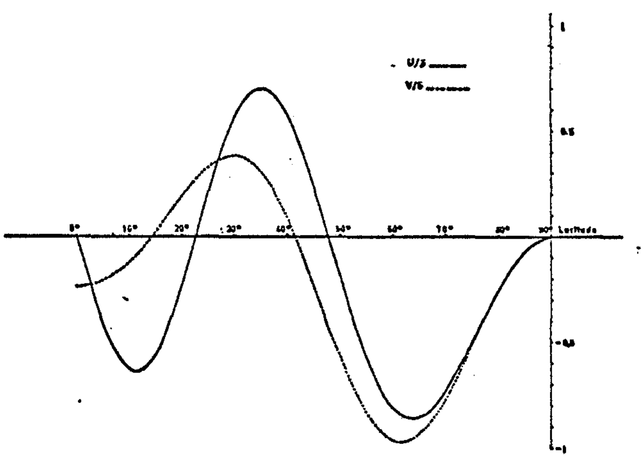

Figure 3.

Variation in latitude of v and u for the third mode of the terdiurnal symmetrical tide.

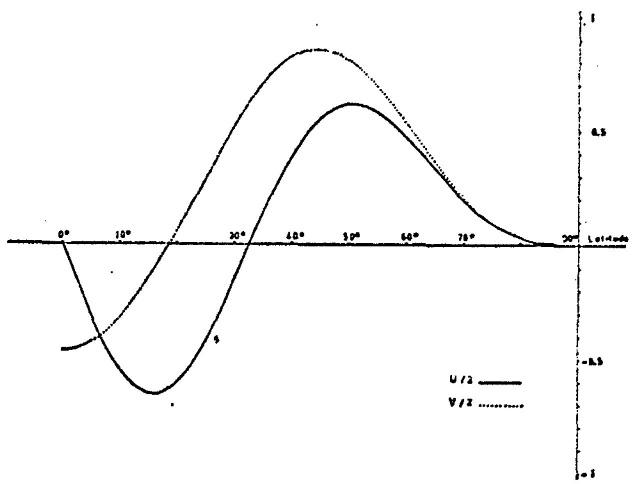

Figure 4.

Variation in latitude of v and u for the second mode of the quartdiurnal symmetrical tide.

noteworthy. For $S^3_{3.5}$ the reverse rotation zone is situated between 22° and 34° of latitude. For $S^3_{3.7}$, there are already two zones; the first between 14° and 23°, and the second between 42° and 48°. In general, the number of reverse /4
rotation zones grows proportionately as we increase the distance from the first mode, but at the same time their extent in latitude becomes narrower.

For the symmetrical tides, the first mode does not have reverse rotation zones; on the other hand, for unsymmetrical tides, there are always some reverse rotation zones. It is necessary to state also that, for the unsymmetrical tides and for all the modes, there is always one reverse rotation zone near the equator. In the table are the reverse rotation zones of principal modes.

TABLE

Reverse Rotation Zones

	Latitudes	
$\Theta^2_{2.4}$	24° to 30°	
$\Theta^2_{2.6}$	15° to 19°	39° to 42°
$\Theta^3_{3.5}$	21° to 34°	
$\Theta^3_{3.7}$	14° to 23°	42° to 48°
$\Theta^4_{4.6}$	18° to 32°	
$\Theta^4_{4.8}$	13° to 22°	38° to 46°
$\Theta^5_{5.7}$	17° to 31°	
$\Theta^5_{5.9}$	11° to 21°	35° to 45°
$\Theta^6_{6.8}$	15° to 29°	
$\Theta^6_{6.10}$	11° to 20°	33° to 42°
$\Theta^2_{2.3}$	0° to 20°	
$\Theta^2_{2.5}$	0° to 12°	33° to 37°
$\Theta^3_{3.4}$	0° to 22°	
$\Theta^3_{3.6}$	0° to 13°	33° to 42°
$\Theta^4_{4.5}$	0° to 21°	
$\Theta^4_{4.7}$	0° to 13°	30° to 40°

2. Rotation in Time

Analysis of the rotation of the velocity vector about a fixed point in space shows that it is defined with no more certainty than for the spatial rotation, since it depends solely on the sign of $f(\theta)$, as described previously, and is independent of the direction of the energy flux.

Beginning with (9) and (10), we can write:

$$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1 \tag{22}$$

where

$$a^2 = A^2 e^x \, |Y(x)|^2 \, [u(\theta)]^2 \tag{23}$$

$$b^2 = A^2 e^x \, |Y(x)|^2 \, [v(\theta)]^2 \tag{24}$$

or else as a function of α

$$V^2 = \frac{a^2 b^2}{b^2 \cos^2\alpha + a^2 \sin^2\alpha} \tag{26}$$

which shows that at a fixed point of space, the velocity vector describes an ellipse whose axes are oriented N-S, E-W and the relationship of the axes is given by $|f(\theta)|$: the direction is described by the sign of $f(\theta)$ as for the preceding case. If we find a diurnal tide which describes an ellipse with the axes not oriented in the N-S, E-W directions, it is a question of mixing no more than one mode.

Conclusion

The detailed study of the direction of rotation of the horizontal velocity vector in the atmospheric tides shows that there is not a characteristic direction of rotation for each hemisphere, but that there are for each mode some zones of latitude where we can define, without ambiguity, the direction of rotation.

We have remarked [Bedinger, Knaflich, Manring, Layzer, 1968], that hodographs exist which turn in the reverse direction, but that, in this case, the modulus of the velocity vector is, in general, smaller if we compare it to other hodographs where the direction of rotation is currently acknowledged.

This is explained if we look at the figures.

The variations in latitude of $v(\theta)$ and $u(\theta)$ correspond, generally, to a divergence which increases from pole to equator. This divergence is never too large, so that we are able to find the maximum absolute value of $v(\theta)$ and $u(\theta)$ in a reverse rotation zone. /5

It is, therefore, normal that in the case of reverse rotation of the hodograph, we find some lower velocities.

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